AP Calculus

Course Description

At the high school level, AP Calculus will be the culmination of your mathematics training. There are limitations to what can be accomplished with algebra and geometry. With concepts you will learn in AP Calculus, the derivative and the integral, many new doors will be opened to you as to what you can do with mathematics. Your training in algebra and geometry will provide you the necessary skills to help you pass through those doors.

Course Objectives

The student will

- Review material from previous coursework essential for the study of calculus.
- Learn the concept of limit, the underlying concept in calculus.
- Learn and study the connection between slope and speed and other quantities of interest.
- Learn to graph and analyze complicated functions as well as solve equations involving these functions.
- Explore the relationships among problems involving finding a function from its derivative, finding the average value of a function and finding the area under a curve and learn a variety of techniques for solving them.
- Discover the versatility of the integral by exploring numerous applications.
- Learn new techniques of integration which provide a broad range of tools used to solve countless problems of interest to engineers, mathematicians and scientists.
- Learn the basic theory and a few common applications of some elementary differential equations.

Primary Textbook

Smith and Minton. *Calculus – Early Transcendental Functions.* Third Edition. McGraw Hill, 2007.

Technology Requirement

Texas Instruments 84 Plus. We will use the calculator in a many different ways, including:

- Conducting explorations
- Graphing functions
- Solving equations numerically
- Analyzing and interpreting results
- Justifying and explaining results of graph and equations

Student Evaluation

As a part of the evaluation process students will be assigned problems which will require them to explain or justify solutions using well-written sentences. Each unit will have one or two exams based upon the amount of material presented. Occasionally included in these exams will be examples of free-response and multiple-choice questions similar to what the student will encounter on the AP exam. There also will be a semester one final.

Unit 1 : Limits and Continuity (approximately 4 weeks)

A. *Student Activity – How Many Fish in the Lake*

This activity is designed to introduce students to the idea of a numerical limit and to help them become more familiar with some of the features of their calculators. With the number of fish decreasing annually by 20%, students will construct tables to find if the number of fish stabilize when the lake is restocked with varying numbers of fish each year. Also, they will be asked which levels of restocking allow the number of fish to reach a desired level. Students will be asked to explain their solution in complete, well-written sentences.

B. The Concept of Limit (including one-sided limits)

1. Determining Limits Graphically
2. Determining Limits Numerically

C. Computation of Limits (including one-sided limits)

1. Finding Limits Using Algebra
2. The Squeeze Theorem
3. Limit of a Piecewise Function

D. Continuity and its Consequences

1. Point, Jump and Infinite Discontinuity
2. Continuous Functions
3. Intermediate Value Theorem

E. Limits Involving Infinity

1. Asymptotes
    a. Vertical
    b. Horizontal
    c. Slant
2. Limits at Infinity

F. *Student Activity – Order of Magnitude of a Function*

This activity demonstrates the relative order of magnitude between two functions using a limit of the ratio of the two. Students, using their graphing calculators, will use both graphical and numerical modes to view the problem. Using different pairs of functions; some exponential, some logarithmic and some polynomial; the students will be asked to list these types of functions in order of increasing magnitude.

Unit 2 : Differentiation (approximately 6 weeks)

A. Tangent Lines and Velocity

1. Tangent Line to a Point on a Curve
2. Difference Quotient
3. Graphical and Numerical Approximation of the Slope of a Tangent Line
4. *Student Activity – The 20 Minute Ride*

   This activity is designed to help students see the need for an instantaneous rate of change, which is the derivative. Using their calculators, the students will complete a table which relates time and distance. The data for this table will be distance traveled (to the nearest 0.1 mile) at the end of each minute during a 20 minute car ride. They will then graph a scatter plot from which they will see the relationship between a distance function and its first derivative (velocity). The students will also calculate average velocity during smaller and smaller time intervals leading to the concept of instantaneous velocity.
5. Average and Instantaneous Velocity
6. Derivatives as a Rate of Change

B. Derivative

1. Derivative as the Limit of the Difference Quotient
2. Differentiability and Continuity
3. Relationship Between the Graphs of f and f'
4. Alternative Derivative Notations
5. Numerical Differentiation

C. Computation of Derivatives : The Power Rule

1. Power Rule

2. General Derivative Rules
    a. Sum
    b. Difference
    c. Multiplying by a Constant
3. Higher Order Derivatives
4. Acceleration

D. Product and Quotient Rules
1. Product
2. Quotient
3. Applications

E. Chain Rule

F. Derivatives of Trigonometric Functions and Applications
1. Derivatives of the Six Trigonometric Functions
2. Applications
    a. Spring-Mass System
    b. Simple Electrical Circuit
3. Equation of a Tangent Line

G. Derivatives of Exponential and Logarithmic Functions
1. Exponential Function
2. Natural Logarithm
3. Logarithmic Differentiation

H. Implicit Differentiation
1. First and Second Derivatives Found Implicitly
2. Derivatives of the Inverse Trigonometric Functions

I. Mean Value Theorem
1. Rolle's Theorem
2. Mean Value Theorem

3. Increasing and Decreasing Functions

Unit 3 : Applications of Differentiation (approximately 5 weeks)

A. Linear Approximations

B. Maximum and Minimum Values

1. Extreme Value Theorem
2. Absolute vs. Local Extrema
3. Definition of a Critical Number

C. Increasing and Decreasing Functions

1. Relationship of the Derivative to the Graph of a Function
2. First Derivative Test
3. Curve Sketching

D. Concavity and the Second Derivative Test

1. Second Derivative and Concavity
2. Inflection Points
3. Second Derivative Test
4. Curve Sketching
5. *Student Activity – Reasoning from a Graph*

   This activity is a card matching game. Color coded cards are provided to the students. One color card displays a function. Another color, a verbal description of the graph of that function. Yet another, the graph of the given function. Finally, third and fourth color cards display the graph of the first and second derivative of said function. The students will play various matching games helping them to see the relationship between functions and their derivatives.

E. Overview of Curve Sketching

F. Optimization

G. Related Rates

H. Rates of Change in Economics and the Sciences

Unit 4 : Integration (approximately 5 weeks)

A. Antiderivatives

1. Indefinite Integral
2. Power Rule
3. Sums and Differences

B. Sums and Sigma Notation

C. Area

1. Approximating Area with Rectangles
2. Computing Area Exactly
3. Riemann Sum
    a. Left Evaluation Point
    b. Right Evaluation Point
    c. Midpoint Evaluation Point
4. *Student Activity – Another 20 Minute Ride*

    Student data gathered earlier in the year for the activity "20 Minute Ride" may be used once again for this activity. This activity is designed to show the students a practical application of Riemann sums. The definite integral has more applications than simply area under a curve. Through questioning the students will be led to see that the definite integral can be used as an accumulator to lead to better and better approximations for distance traveled.

D. Definite Integral

1. Using Riemann Sums to Compute a Definite Integral
2. Signed vs. Total Area
3. *Student Activity – What's an Accumulator?*

    Yet another activity designed to reinforce the concept of the definite integral as an accumulator. Students will be asked to sketch graphs dealing with topics such as daily television set sales, velocity, high school graduates, birth rate, price of a cup of coffee and rate of change of temperature; write Riemann sums for the given relationships and write a definite integral to represent the relationship.

4. Average Value of a Function

5. Integral Mean Value Theorem

E. Fundamental Theorem of Calculus

1. Part 1
2. Part 2

F. Integration by Substitution

1. Indefinite Integrals
2. Definite Integrals

G. Numerical Integration

1. Midpoint Rule
2. Trapezoidal Rule
3. Estimating an Integral from a Table of Function Values

H. Natural Logarithm as an Integral

1. Properties of Logarithms
2. Logarithmic Differentiation
3. Exponential Function

Unit 5 : Applications of the Definite Integral (approximately 5 weeks)

A. Area Between Curves

1. Integrating with Respect to *x*
2. Integrating with Respect to *y*

B. Volume : Slicing, Disks and Washers

1. Slicing
2. Disks
3. Washers

C. Projectile Motion

D. Applications of Integration to Physics and Engineering

1. Work
2. Impulse
3. Mass

4. Hydrostatic Force

E. Probability

1. Probability Density Function
2. Mean and Median

Unit 6 : Integration Techniques (approximately 3 weeks)

A. Review of Formulas and Techniques

B. Trigonometric Techniques of Integration

1. *u* Substitution
2. Trigonometric Substitution

Unit 7 : First-Order Differential Equations (approximately 2 weeks)

A. Modeling with Differential Equations

1. Growth and Decay Problems
2. Compound Interest

B. Separable Differential Equations

1. Solving Separable Differential Equations
2. Initial Value Problems

C. Slope Fields

1. Constructing Slope Fields
2. Using a Slope Field to Visualize the Behavior of Solutions

Unit 8 : Review and Test Preparation (approximately 4 weeks)

A. Multiple-Choice Practice

B. Free-Response Practice

Unit 9 : Non AP Topics (approximately 2 weeks)

A. L'Hôpital's Rule

B. Simpson's Rule

C. Integration by Parts